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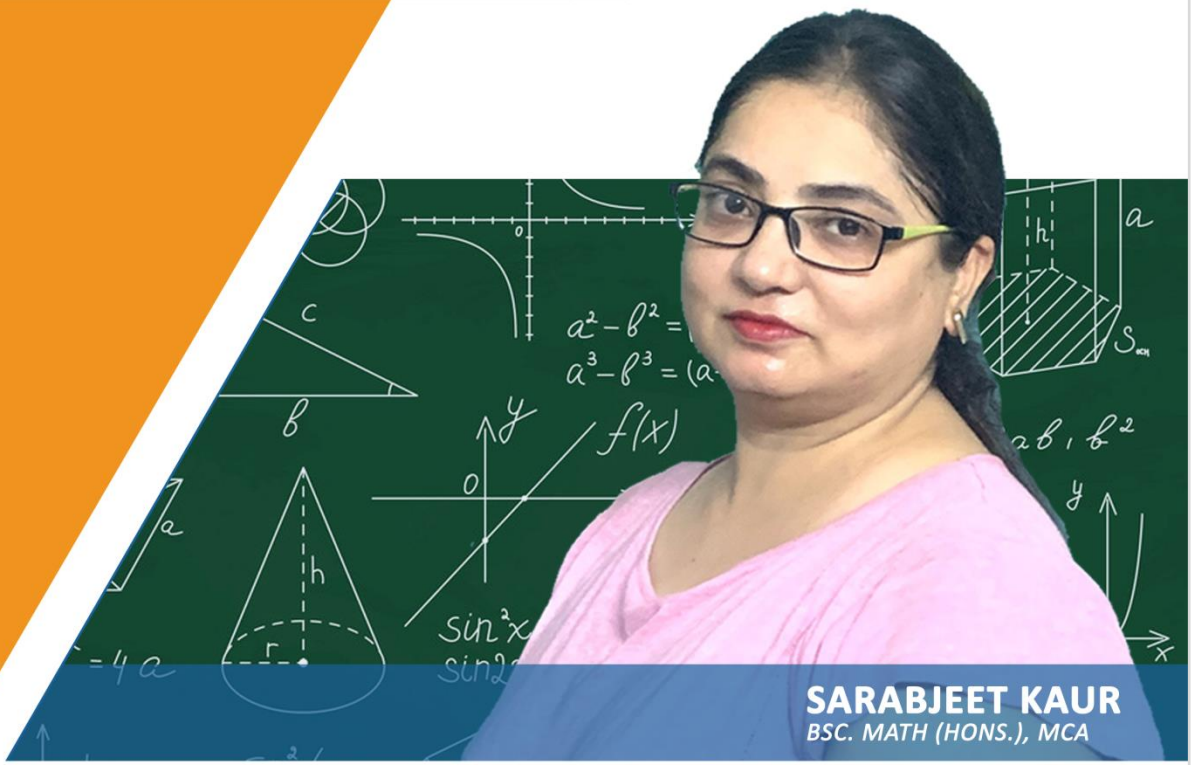

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BSC. MATH (HONS.), MCA

GRADE 12TH MATHS

CHAPTER 1

# Relation and Functions

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*Mark (✓) against the correct answer in each of the following:*

1. $f : N \to N : f(x) = 2x$ is
   (a) one-one and onto
   (b) one-one and into
   (c) many-one and onto
   (d) many-one and into

2. $f : N \to N : f(x) = x^2 + x + 1$ is
   (a) one-one and onto
   (b) one-one and into
   (c) many-one and onto
   (d) many-one and into

3. $f : R \to R : f(x) = x^2$ is
   (a) one-one and onto
   (b) one-one and into
   (c) many-one and onto
   (d) many-one and into

4. $f : R \to R : f(x) = x^3$ is
   (a) one-one and onto
   (b) one-one and into
   (c) many-one and onto
   (d) many-one and into

5. $f : R^+ \to R^+ : f(x) = e^x$ is
   (a) many-one and into
   (b) many-one and onto
   (c) one-one and into
   (d) one-one and onto

6. $f : \left[\frac{-\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1] : f(x) = \sin x$ is
   (a) one-one and into
   (b) one-one and onto
   (c) many-one and into
   (d) many-one and onto

7. $f : R \to R : f(x) = \cos x$ is
   (a) one-one and into
   (b) one-one and onto
   (c) many-one and into
   (d) many-one and onto

8. $f : C \to R : f(z) = |z|$ is
   (a) one-one and into
   (b) one-one and onto
   (c) many-one and into
   (d) many-one and onto

9. Let $A = R - \{3\}$ and $B = R - \{1\}$. Then, $f : A \to B : f(x) = \dfrac{(x-2)}{(x-3)}$ is

(a) one-one and into
(b) one-one and onto
(c) many-one and into
(d) many-one and onto

10. Let $f : N \to N : f(n) = \begin{cases} \dfrac{1}{2}(n+1), & \text{when } n \text{ is odd} \\ \dfrac{n}{2}, & \text{when } n \text{ is even.} \end{cases}$

Then, $f$ is

(a) one-one and into
(b) one-one and onto
(c) many-one and into
(d) many-one and onto

11. Let $A$ and $B$ be two non-empty sets and let $f : (A \times B) \to (B \times A) : f(a, b) = (b, a)$. Then, $f$ is

(a) one-one and onto
(b) one-one and into
(c) many-one and onto
(d) many-one and into

12. Let $f : Q \to Q : f(x) = (2x + 3)$. Then, $f^{-1}(y) = ?$

(a) $(2y - 3)$
(b) $\dfrac{1}{(2y-3)}$
(c) $\dfrac{1}{2}(y - 3)$
(d) none of these

13. Let $f : R - \left\{\dfrac{-4}{3}\right\} \to R - \left\{\dfrac{4}{3}\right\} : f(x) = \dfrac{4x}{(3x+4)}$. Then, $f^{-1}(y) = ?$

(a) $\dfrac{4y}{(4-3y)}$
(b) $\dfrac{4y}{(4y+3)}$
(c) $\dfrac{4y}{(3y-4)}$
(d) none of these

14. Let $f : N \to X : f(x) = 4x^2 + 12x + 15$. Then, $f^{-1}(y) = ?$

(a) $\dfrac{1}{2}(\sqrt{y-4} + 3)$
(b) $\dfrac{1}{2}(\sqrt{y-6} - 3)$
(c) $\dfrac{1}{2}(\sqrt{y-4} + 5)$
(d) none of these

15. If $f(x) = \dfrac{(4x + 3)}{(6x - 4)}$, $x \neq \dfrac{2}{3}$ then $(f \circ f)(x) = ?$

(a) $x$ (b) $(2x - 3)$ (c) $\dfrac{4x - 6}{3x + 4}$ (d) none of these

16. If $f(x) = (x^2 - 1)$ and $g(x) = (2x + 3)$ then $(g \circ f)(x) = ?$

(a) $(2x^2 + 3)$ (b) $(3x^2 + 2)$ (c) $(2x^2 + 1)$ (d) none of these

17. If $f\left(x + \dfrac{1}{x}\right) = x^2 + \dfrac{1}{x^2}$ then $f(x) = ?$

(a) $x^2$ (b) $(x^2 - 1)$ (c) $(x^2 - 2)$ (d) none of these

18. If $f(x) = \dfrac{1}{(1 - x)}$ then $(f \circ f \circ f)(x) = ?$

(a) $\dfrac{1}{(1 - 3x)}$ (b) $\dfrac{x}{(1 + 3x)}$ (c) $x$ (d) none of these

19. If $f(x) = \sqrt[3]{3 - x^3}$ then $(f \circ f)(x) = ?$

(a) $x^{1/3}$ (b) $x$ (c) $(1 - x^{1/3})$ (d) none of these

20. If $f(x) = x^2 - 3x + 2$ then $(f \circ f)(x) = ?$

(a) $x^4$ (b) $x^4 - 6x^3$ (c) $x^4 - 6x^3 + 10x^2$ (d) none of these

21. If $f(x) = 8x^3$ and $g(x) = x^{1/3}$ then $(g \circ f)(x) = ?$

(a) $x$ (b) $2x$ (c) $\dfrac{x}{2}$ (d) $3x^2$

22. If $f(x) = x^2$, $g(x) = \tan x$ and $h(x) = \log x$ then $\{h \circ (g \circ f)\}\left(\sqrt{\dfrac{\pi}{4}}\right) = ?$

(a) 0 (b) 1 (c) $\dfrac{1}{x}$ (d) $\dfrac{1}{2}\log\dfrac{\pi}{4}$

23. If $f = \{(1, 2), (3, 5), (4, 1)\}$ and $g = \{(2, 3), (5, 1), (1, 3)\}$ then $(g \circ f) = ?$

(a) $\{(3, 1), (1, 3), (3, 4)\}$ (b) $\{(1, 3), (3, 1), (4, 3)\}$

(c) $\{(3, 4), (4, 3), (1, 3)\}$ (d) $\{(2, 5), (5, 2), (1, 5)\}$

24. Let $f(x) = \sqrt{9 - x^2}$. Then, dom $(f) = ?$

(a) $[-3, 3]$ (b) $(-\infty, -3]$
(c) $[3, \infty)$ (d) $(-\infty, -3] \cup (4, \infty)$

25. Let $f(x) \sqrt{\frac{x-1}{x-4}}$. Then, dom $(f) = ?$

(a) $[1, 4)$ (b) $[1, 4]$
(c) $(-\infty, 4]$ (d) $(-\infty, 1] \cup (4, \infty)$

26. Let $f(x) = e^{\sqrt{x^2-1}} \cdot \log(x - 1)$. Then, dom $(f) = ?$

(a) $(-\infty, 1]$ (b) $[-1, \infty)$
(c) $(1, \infty)$ (d) $(-\infty, -1] \cup (1, \infty)$

27. Let $f(x) = \frac{x}{(x^2 - 1)}$. Then, dom $(f) = ?$

(a) $R$ (b) $R - \{1\}$ (c) $R - \{-1\}$ (d) $R - \{-1, 1\}$

28. Let $f(x) = \frac{\sin^{-1} x}{x}$. Then, dom $(f) = ?$

(a) $(-1, 1)$ (b) $[-1, 1]$ (c) $[-1, 1] - \{0\}$ (d) none of these

29. Let $f(x) = \cos^{-1} 2x$. Then, dom $(f) = ?$

(a) $[-1, 1]$ (b) $\left[\frac{-1}{2}, \frac{1}{2}\right]$ (c) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ (d) $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

30. Let $f(x) = \cos^{-1}(3x - 1)$. Then, dom $(f) = ?$

(a) $\left(0, \frac{2}{3}\right)$ (b) $\left[0, \frac{2}{3}\right]$ (c) $\left[\frac{-2}{3}, \frac{2}{3}\right]$ (d) none of these

31. Let $f(x) = \sqrt{\cos x}$. Then, dom $(f) = ?$

(a) $\left[0, \frac{\pi}{2}\right]$ (b) $\left[\frac{3\pi}{2}, 2\pi\right]$
(c) $\left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$ (d) none of these

32. Let $f(x) = \sqrt{\log(2x - x^2)}$. Then, dom $(f) = ?$

(a) $(0, 2)$ (b) $[1, 2]$ (c) $(-\infty, 1]$ (d) none of these

33. Let $f(x) = x^2$. Then, dom $(f)$ and range $(f)$ are respectively

(a) $R$ and $R$ (b) $R^+$ and $R^+$ (c) $R$ and $R^+$ (d) $R$ and $R - \{0\}$

34. Let $f(x) = x^3$. Then, dom $(f)$ and range $(f)$ are respectively

(a) $R$ and $R$ (b) $R^+$ and $R^+$
(c) $R$ and $R^+$ (d) $R^+$ and $R$

35. Let $f(x) = \log(1 - x) + \sqrt{x^2 - 1}$. Then, dom $(f) = ?$

(a) $(1, \infty)$ (b) $(-\infty, -1]$ (c) $[-1, 1)$ (d) $(0, 1)$

36. Let $f(x) = \dfrac{1}{(1 - x^2)}$. Then, range $(f) = ?$

(a) $(-\infty, 1]$ (b) $[1, \infty)$ (c) $[-1, 1]$ (d) none of these

37. Let $f(x) = \dfrac{x^2}{(1 + x^2)}$. Then, range $(f) = ?$

(a) $[1, \infty)$ (b) $[0, 1)$ (c) $[-1, 1]$ (d) $(0, 1]$

38. The range of $f(x) = x + \dfrac{1}{x}$ is

(a) $[-2, 2]$ (b) $[2, \infty)$ (c) $(-\infty, -2]$ (d) none of these

39. The range of $f(x) = a^x$, where $a > 0$ is

(a) $]-\infty, 0]$ (b) $]-\infty, 0)$ (c) $[0, \infty)$ (d) $(0, \infty)$

## ANSWERS (OBJECTIVE QUESTIONS)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1.** (b) | **2.** (b) | **3.** (d) | **4.** (a) | **5.** (d) | **6.** (b) | **7.** (c) | **8.** (c) | **9.** (b) | **10.** (d) |
| **11.** (a) | **12.** (c) | **13.** (a) | **14.** (b) | **15.** (a) | **16.** (c) | **17.** (c) | **18.** (c) | **19.** (b) | **20.** (d) |
| **21.** (b) | **22.** (a) | **23.** (b) | **24.** (a) | **25.** (d) | **26.** (c) | **27.** (d) | **28.** (c) | **29.** (b) | **30.** (b) |
| **31.** (c) | **32.** (d) | **33.** (c) | **34.** (a) | **35.** (b) | **36.** (b) | **37.** (b) | **38.** (d) | **39.** (d) | |

## HINTS TO SOME SELECTED OBJECTIVE QUESTIONS

**1.** $2x = 3 \Rightarrow x = \dfrac{3}{2} \notin N$. So, $f$ is into.

**2.** $f(x_1) = f(x_2) \Rightarrow x_1^2 + x_1 + 1 = x_2^2 + x_2 + 1$

$\Rightarrow (x_1^2 - x_2^2) + (x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(x_1 + x_2 + 1) = 0$

$\Rightarrow x_1 - x_2 = 0 \Rightarrow x_1 = x_2$.

$\therefore$ $f$ is one-one.

$f(x) = 1 \Rightarrow x^2 + x + 1 = 1 \Rightarrow x(x + 1) = 0 \Rightarrow x = 0$ or $x = -1$.

And, none of 0 and $-1$ is in $N$. So, $f$ is into.

**5.** $f(x_1) = f(x_2) \Rightarrow e^{x_1} = e^{x_2} \Rightarrow x_1 = x_2$. So, $f$ is one-one.

For each $x \in R^+$ $\exists \log x \in R^+$ s.t. $f(\log x) = x$.

So, $f$ is onto.

7. $\cos(2\pi - \theta) = \cos\theta \Rightarrow f$ is many-one.

Range $(f) = [-1, 1] \subset R \Rightarrow f$ is into.

**8.** $i \neq -i$. But $f(i) = f(-i) = 1$. So, $f$ is many-one.

$-1 \in R$ having no pre-image in $C$. So, $f$ is into.

**9.** $f(x_1) = f(x_2) \Rightarrow \dfrac{(x_1 - 2)}{(x_1 - 3)} = \dfrac{(x_2 - 2)}{(x_3 - 3)} \Rightarrow x_1 = x_2$. So, $f$ is one-one.

Let $\dfrac{x-2}{x-3} = y$. Then, $x = \dfrac{3y-2}{y-1}$. Clearly, $y \neq 1$ and $x \neq 3$.

$\therefore$ $f(x) = y$ and so $f$ is onto.

**10.** $f(1) = f(2)$ shows that $f$ is many-one.

If $n$ is odd then $(2n - 1)$ is odd and $f(2n - 1) = n$.

If $n$ is even then $2n$ is even and $f(2n) = n$.

$\therefore$ $f$ is onto.

**12.** $y = 2x + 3 \Rightarrow x = \dfrac{1}{2}(y - 3) \Rightarrow f^{-1}(y) = \dfrac{1}{2}(y - 3)$.

**13.** $y = \dfrac{4x}{3x+4} \Rightarrow x = \dfrac{4y}{(4 - 3y)} \Rightarrow f^{-1}(y) = \dfrac{4y}{(4 - 3y)}$.

**14.** $y = 4x^2 + 12x + 15 = (2x + 3)^2 + 6 \Rightarrow x = \dfrac{1}{2}(\sqrt{y-6} - 3)$

$\therefore$ $f^{-1}(y) = \dfrac{1}{2}(\sqrt{y-6} - 3)$.

**15.** $f(x) = \dfrac{4x+3}{6x-4} = y$ (say).

$$\text{Then } f(y) = \frac{4y+3}{6y-4} = \frac{4\left(\dfrac{4x+3}{6x-4}\right)+3}{6\left(\dfrac{4x+3}{6x-4}\right)-4} = \frac{34x}{34} = x.$$

$\Rightarrow$ $f[f(x)] = x \Rightarrow (f \circ f)(x) = x$.

**16.** $(g \circ f)(x) = g[f(x)] = g(x^2 - 1)$

$= 2(x^2 - 1) + 3 = (2x^2 + 1)$.

**17.** Let $x + \dfrac{1}{x} = z$. Then,

$$f(z) = f\left(x + \frac{1}{x}\right) = \left(x^2 + \frac{1}{x^2}\right) = \left(x + \frac{1}{x}\right)^2 - 2 = (z^2 - 2).$$

$\Rightarrow$ $f(x) = (x^2 - 2)$.

**18.** $(f \circ f)(x) = f\{f(x)\} = f\left(\dfrac{1}{1-x}\right) = \dfrac{1}{\left(1 - \dfrac{1}{1-x}\right)} = \dfrac{1-x}{-x} = \dfrac{x-1}{x}$

$$\Rightarrow \{f \circ (f \circ f)\}(x) = f\{(f \circ f)(x)\} = f\left(\frac{x-1}{x}\right) = \frac{1}{1 - \dfrac{x-1}{x}} = x.$$

**19.** $(f \circ f)(x) = f\{f(x)\} = \{(3 - x^3)^{1/3}\} = f(y)$, where $y = (3 - x^3)^{1/3}$

$= (3 - y^3)^{1/3} = [3 - \{3 - x^3\}]^{1/3} = (x^3)^{1/3} = x$.

**20.** $(f\,o\,f)(x) = f\{f(x)\} = f(x^2 - 3x + 2) = f(y)$, where $y = (x^2 - 3x + 2)$

$= y^2 - 3y + 2 = (x^2 - 3x + 2)^2 - 3(x^2 - 3x + 2) + 2$

$= (x^4 - 6x^3 + 10x^2 - 3x)$.

**21.** $(g\,o\,f)(x) = g[f(x)] = g(8x^3) = (8x^3)^{1/3} = 2x$.

**22.** $\{h\,o\,(g\,o\,f)\}(x) = (h\,o\,g)\{f(x)\} = (h\,o\,g)(x^2)$

$= h\{g(x^2)\} = h(\tan x^2) = \log(\tan x^2)$.

$\therefore \quad \{h\,o\,(g\,o\,f)\}\sqrt{\frac{\pi}{4}} = \log\left(\tan\frac{\pi}{4}\right) = \log 1 = 0$.

**23.** Dom $(g\,o\,f)$ = dom $(f) = \{1, 3, 4\}$.

$(g\,o\,f)(1) = g\{f(1)\} = g(2) = 3, (g\,o\,f)(3) = g\{f(3)\} = g(5) = 1$

$(g\,o\,f)(4) = g\{f(4)\} = g(1) = 3$

$\therefore \quad g\,o\,f = \{(1,3), (3,1), (4,3)\}$.

**24.** $f(x)$ is defined only when $9 - x^2 \geq 0 \Rightarrow x^2 \leq 9 \Rightarrow -3 \leq x \leq 3$.

$\therefore \quad$ dom $(f) = [-3, 3]$.

**25.** $f(x)$ is defined when $x - 4 \neq 0$ and $\frac{x-1}{x-4} \geq 0$

$\Rightarrow \quad x \neq 4$ and $(x \geq 4$ or $x \leq 1) \Rightarrow (x > 4$ or $x \leq 1)$

$\Rightarrow \quad$ dom $(f) = (-\infty, 1] \cup (4, \infty)$.

**26.** $f(x)$ is defined only when $(x^2 - 1) \geq 0$ and $(x - 1) > 0$

$\Rightarrow \quad (x - 1)(x + 1) \geq 0$ and $(x - 1) > 0 \Rightarrow x + 1 \geq 0$ and $x - 1 > 0 \Rightarrow x > 1$

$\therefore \quad$ dom $(f) = (1, \infty)$.

**27.** $f(x)$ is not defined when $(x^2 - 1) = 0$, i.e., when $(x - 1)(x + 1) = 0$,

i.e., when $x = 1$ or $x = -1$.

$\therefore \quad$ dom $(f) = R - \{1, -1\}$.

**28.** $\frac{\sin^{-1} x}{x}$ is defined only when $x \neq 0$ and $x \in [-1, 1]$.

$\therefore \quad$ dom $(f) = [-1, 1] - \{0\}$.

**29.** $\sin^{-1} 2x$ is defined only when $2x \in [-1, 1] \Rightarrow x \in \left[\frac{-1}{2}, \frac{1}{2}\right]$.

**30.** $\cos^{-1}(3x - 1)$ is defined only when $(3x - 1) \in [-1, 1]$

$\Rightarrow \quad 3x \in [0, 2] \Rightarrow x \in \left[0, \frac{2}{3}\right] \Rightarrow$ dom $(f) = \left[0, \frac{2}{3}\right]$.

**31.** $f(x)$ is defined only when $\cos x \geq 0$

$\Rightarrow \quad x$ lies in 1st or 4th quadrant

$\Rightarrow \quad$ dom $(f) = \left[0, \frac{\pi}{2}\right] \cup \left[\frac{3\pi}{2}, 2\pi\right]$.

**32.** $f(x)$ is defined only when $\log(2x - x^2) \geq 0$

$\Rightarrow (2x - x^2) \geq 1 \Rightarrow (1 + x^2 - 2x) \leq 0 \Rightarrow (1 - x)^2 \leq 0 \Rightarrow (1 - x) = 0 \Rightarrow x = 1.$

$\therefore$ dom $(f) = \{1\}$.

**33.** $f(x) = x^2$ is clearly defined for each $x \in R$. So, dom $(f) = R$.

$y = x^2 \Rightarrow x = \pm\sqrt{y}$.

When $y < 0$, there is no real value of $x$. So, $y \geq 0$.

$\therefore$ range $(f) = R^+$.

**34.** $f(x) = x^3$ is defined for each $x \in R$. So, dom $(f) = R$.

For each $y \in R$, $y^{1/3} \in R$ and so $x = y^{1/3}$ is real.

$\therefore$ range $(f) = R$.

**35.** Let $f(x) = g(x) + h(x)$, where $g(x) = \log(1 - x)$ and $h(x) = \sqrt{x^2 - 1}$.

$g(x)$ is defined only when $1 - x > 0 \Rightarrow x < 1$. So, dom $(g) = (-\infty, 1)$.

$h(x)$ is defined only when $x^2 - 1 \geq 0 \Rightarrow x \geq 1$ or $x \leq -1$.

$\therefore$ dom $(h) = (-\infty, -1] \cup [1, \infty)$.

$\therefore$ dom $(f)$ = dom $(g) \cap$ dom $(h) = (-\infty, -1]$.

**36.** $y = \dfrac{1}{(1 - x^2)} \Rightarrow x = \sqrt{1 - \dfrac{1}{y}}$.

Clearly, $x$ is not defined when $y = 0$ or $1 - \dfrac{1}{y} < 0$, i.e., $y = 0$ or $y < 1$.

$\therefore$ range $(f) = [1, \infty)$.

**37.** $y = \dfrac{x^2}{(1 + x^2)} \Rightarrow x = \sqrt{\dfrac{y}{1 - y}}$.

Clearly, $x$ is defined only when $\dfrac{y}{(1 - y)} \geq 0$, and $(1 - y) \neq 0$, i.e., when $0 \leq y < 1$.

So, range $(f) = [0, 1)$.

**38.** $y = \dfrac{x^2 + 1}{x} \Rightarrow x^2 - xy + 1 = 0 \Rightarrow x = \dfrac{y \pm \sqrt{y^2 - 4}}{2}$.

$x$ is defined when $(y^2 - 4) \geq 0 \Rightarrow y^2 \geq 4 \Rightarrow y \geq 2$ or $y \leq -2$.

$\therefore$ range $(f) = (-\infty, -2] \cup [2, \infty)$.

**39.** Clearly, $a^x > 0$ whatever may be the value of $x$.

$\therefore$ range $(f) = (0, \infty)$.